

Animated Visualization of Convolution and Correlation

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ABSTRACT: The continuous-time operations “convolution” and “correlation” are basic operations in the field of signals and systems. We made the experience that it is rather difficult for students to understand these operations when explaining these processes by using only conventional, static tools, e.g. a sequence of diagrams on transparencies. So we decided to develop a new tool in order to help the students in understanding these basic signal processing operations.

The project had three objectives: the first was to get a tool which helps explaining the operations during the lectures, the second to give to the students an interactive hand-out with which they can “play” at home and the third was to let other universities also take advantage of it.

INTRODUCTION

In the field of electrical and electronics engineering education there are the fundamental operations of *convolution* and *correlation* which have to be well understood by the students. At our university the operation of convolution is taught in conjunction with linear and time-invariant systems in an undergraduate “signals and systems”-course and the concept of correlation is introduced in telecommunications engineering lectures.

In the past the chalkboard and/or transparencies were used to explain these operations. Of course the use of a computer allows a much better visualization due to its animation capabilities – it’s like the difference between pictures and movies. Moreover in the meantime almost every student owns a computer so it makes sense to give the tool to the students to let them “play around” to understand the operations.

So we decided to develop a software tool for the animated visualization of the above-mentioned operations. Because sometimes students of our postgraduate course (fully taught in English) have to refresh their knowledge of convolution and correlation, and because sometimes there are French students from a partner university, we decided to offer the user interface in German, English and French language. We did the same for the comprehensive help features, so the downloadable and self-explaining tool may be used by every student.

MOTIVATION

To understand the need of such a tool, it is necessary to first have a closer look at the corresponding operations:

The *convolution* of two signals in the time domain is a rather fundamental, but troublesome operation. Actually its difficulty is the main reason for the application of frequency

transformations to circumvent the convolution operation by an easy-to-apply algebraic multiplication. Nevertheless it is a must for the professor to teach and visualize this fundamental operation as well as for the student to eventually understand it.

If we restrict ourselves to continuous-time systems, then the convolution operation is actually a convolution integral [1]:

$$z(t) = x(t) * y(t) = \int_{-\infty}^{+\infty} x(\tau) \cdot y(t - \tau) d\tau$$

τ is the integration-variable and both functions depend on it, except for the fact that y depends on the transformed independent variable $(t - \tau)$. So to get a graphical impression of the result of the parameterized integral at some value of time t , the following steps have to be performed:

1. $x(t) \quad x(\tau)$:
Sketch x versus independent variable τ instead of t .
2. $y(t) \quad y(-\tau)$:
Sketch y versus independent variable τ instead of t and mirror it at the vertical axis.
3. $y(-\tau) \quad y(t-\tau)$:
If $t > 0$, shift $y(-\tau)$ by t to the right,
else shift $y(-\tau)$ by $|t|$ to the left.
4. $p(\tau, t) = x(\tau) \cdot y(t - \tau)$:
Multiply the signals to get the product function $p(\tau, t)$.
5. $z(t) = \int p(\tau, t) d\tau$:
Integrate the product function.

It should be remembered that almost all these operations have to be done for each value of time t being of relevance. Drawing the results of the convolution for different successive instants in a chart reveals that the two input functions move relatively to each other. Although we know this fact it is still hard to imagine the resulting output signal of even simple input signals.

The *cross-correlation* of two deterministic finite-energy signals $x(t)$ and $y(t)$ is very similar to the convolution:

$$\phi_{xy}(\tau) = \int_{-\infty}^{+\infty} x(t + \tau) \cdot y(t) dt$$

In short, the main difference between the two operations is, that for the cross-correlation none of the input signals has to be mirrored at the vertical axis before shifting it. So the result of cross-correlating two signals is commonly easier to interpret and understand than that of convolving them. The correlation is a measure of how much the two signals linearly depend on each other, in other words how strong they correlate.

The *auto-correlation* function represents the special case of correlating a signal $x(t)$ with itself:

$$\phi_{xx}(\tau) = \int_{-\infty}^{+\infty} x(t + \tau) \cdot x(t) dt$$

The result informs about the time-dependence of the values. The auto-correlation function always shows a peak in the origin where the signal has the most resemblance with itself as it is identical.

The fact, that these things are difficult to understand and therefore are difficult to explain and the lack of any similar tool gave us the motivation to develop a program, that lets one easily view at one glance which can not be said better in hundreds of words.

The demand was that the tool should be useable without any knowledge of the context, i.e. it should be self-explaining and in the case of questions there should be a comprehensive help available in German, English and French.

TECHNICAL DESCRIPTION

The program has been written in the C++ programming language. After starting in the *Edit Mode* signals can be entered by using the mouse. With every click of the left mouse-button a point to the signal is added which in turn can be removed by using the "undo"-button. While the mouse-button is held down, all points on the path of the mouse-pointer will be added if the mouse is moved. Of course for reasons of definition a point can never be located left to any point preceding.

If the field "paint diracs" is enabled, every click with the left mouse-button adds a dirac impulse. Its height represents its weight. If occasionally a dirac impulse is misplaced, it can be removed with the "remove last dirac"-button. If one wants to restart entering a new signal, the button "delete signal" will entirely remove the actually displayed signal without saving it.

A grid of 0.5 units can be enabled in the display-menu. It allows to enter signals (especially if they have rectangular shapes) much more precisely. By default the option "snap to

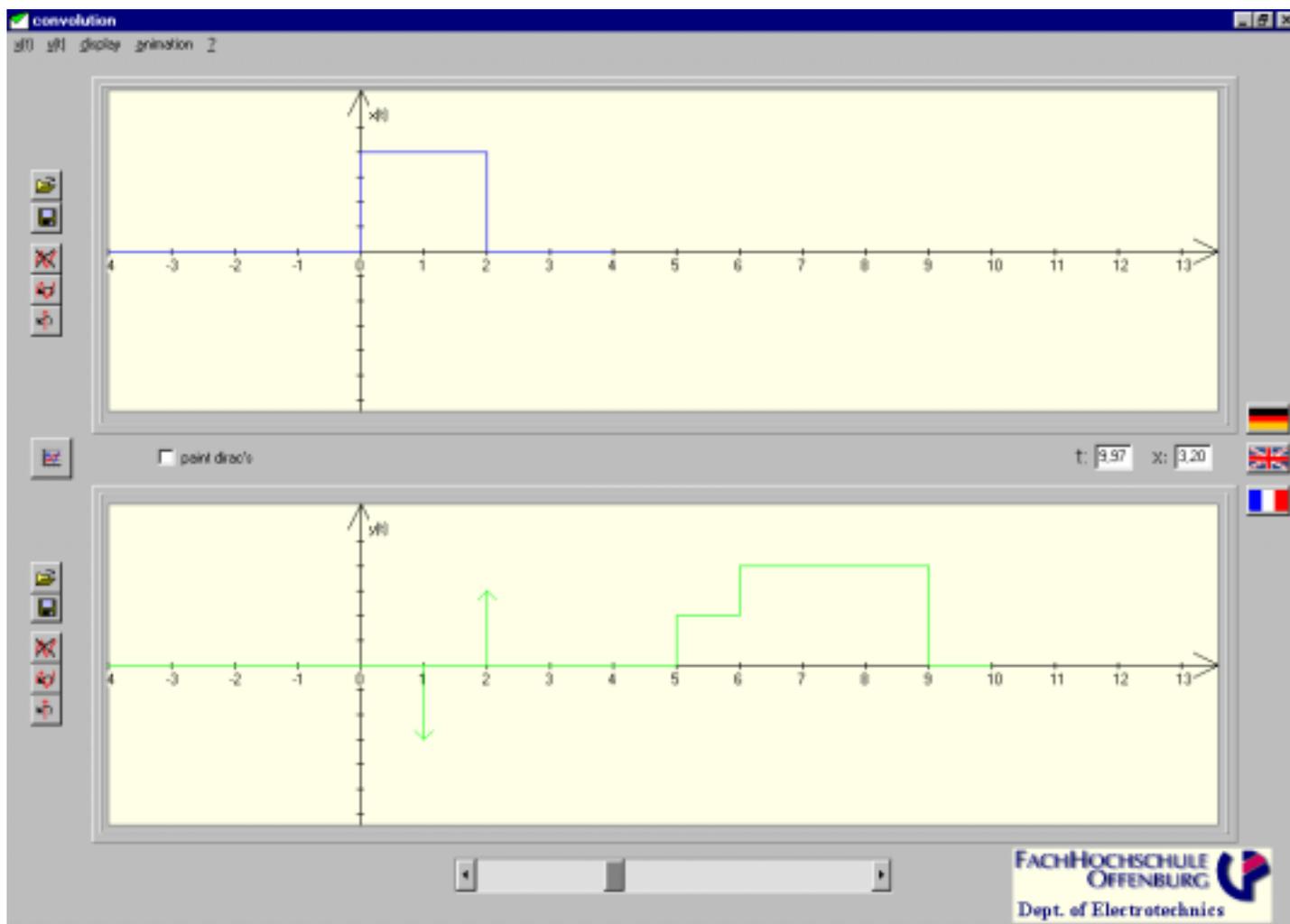


Figure 1: The Edit Mode

grid" will be also activated, which means that the nearest point in the grid to the clicked one will be entered. If "snap to grid" is activated, only single points can be entered. For better orientation there are two fields on the right showing the coordinates of the point which will be added if the left mouse-button is clicked. The colours of the signals can be modified by choosing the option "colour..." in the menu $x(t)$, $y(t)$ or animation.

A click on the big button on the left starts the calculation and changes into the *Animation Mode*.

Now the two input-signals $x(t)$ and $y(t)$ are displayed in the upper chart. In the lower chart the result of the operation is displayed. Initially this chart is left blank. With a click on the play-button the animation is started with the selected speed. A click on "pause" freezes it immediately. "Stop" draws the complete result and resets $y(t)$ to its initial position.

By default the area under the resulting signal is shown hatched. This option can be disabled in the display-menu. Also in the display-menu a grid can be enabled. Between the two charts the name of the actually chosen and calculated operation is displayed.

SUMMARY

We developed a software tool which we regard as being very useful for educational purposes to visualize the operations of convolution and correlation in an animated manner. It may be used by every interested person by downloading it from

www.fh-offenburg.de/e/dsvlab/convolution.htm

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REFERENCE

1. Oppenheim, A.V. and Willsky, A.S., *Signals and Systems (2nd edition)*, Prentice-Hall International Inc., 90-102 (1997).

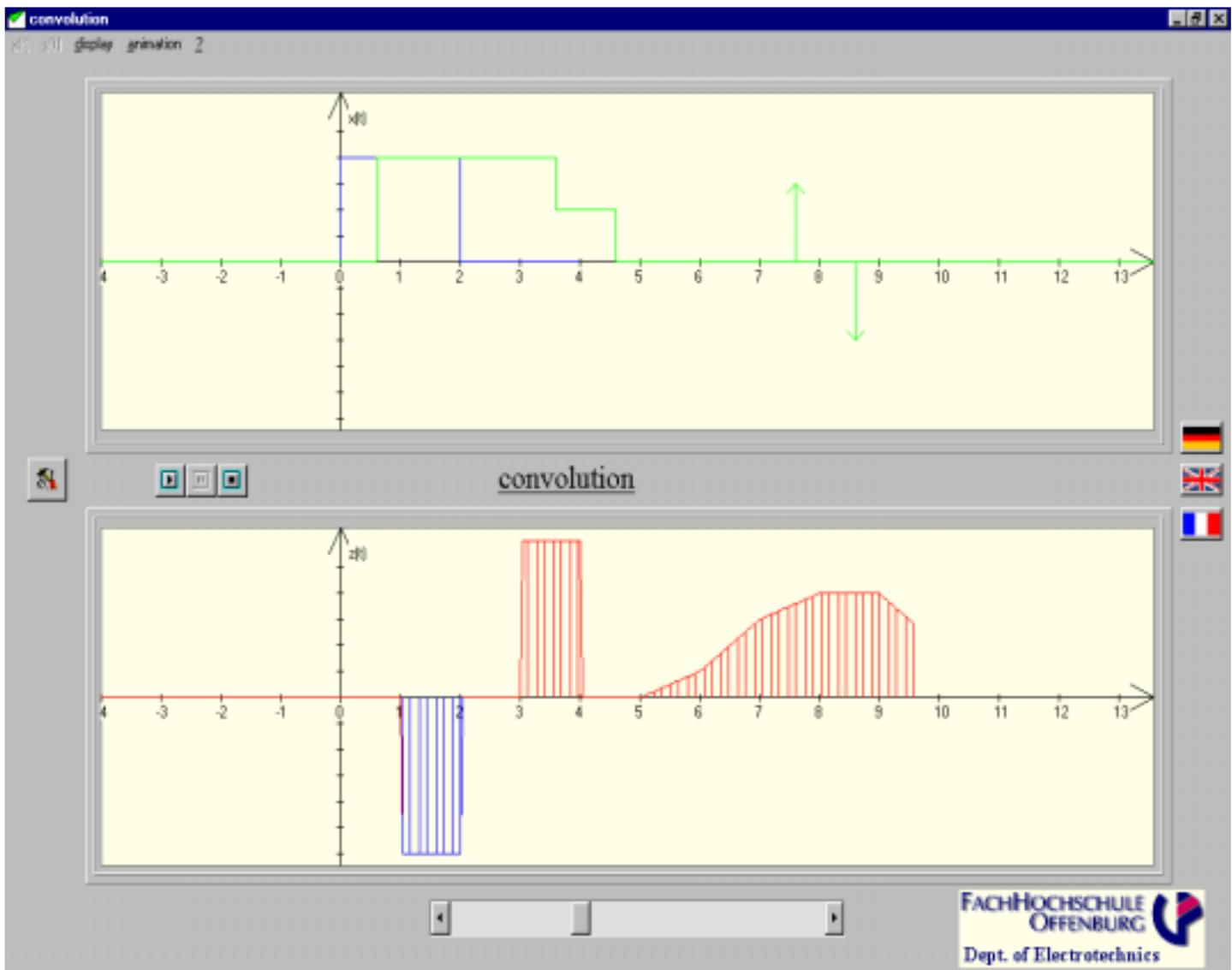


Figure 2: The Animation Mode